

For more information on Texas A&M University's new syllabus requirements, see here.

Course Information

Course Number: MATH 689

Course Title: Topics in Physics for Mathematicians

Section: Not known yet
Time: Not known yet
Location: Not known yet

Credit Hours: 3

Instructor Details

Instructor: Igor Zelenko
Office: Blocker 601J
Phone: 979820-0620

E-Mail: zelenko@math.tamu.edu

Office Hours: will be announced

Course Description

Throughout history, mathematics and physics have always had a fruitful interaction. Many important mathematical research areas, such as symplectic geometry, Hamiltonian dynamics, calculus of variation, differential geometry and Mathematical Gauge Theory, theory of Lie groups, Lie algebras, Clifford algebras, Hopf algebras/ quantum groups, and their representations, spectral theory of differential operators, theory of operator algebras, theory of functional integrals, and algebraic topology (such as theory of characteristic classes of vector bundles and knot theory) were motivated by physical theories. In the opposite direction, physicists have used previously developed mathematical theories, often purely abstract ones or with completely different original motivation, to derive new physical theories (the most famous example is Einstein's use of differential geometry in developing general relativity).

Nonetheless, due to different backgrounds, research goals, and rigorousness standards, mathematicians and physicists frequently fail to understand each other and as a results, many mathematical theories that were originally motivated by physics have strayed from their original motivations and are now purely mathematical in nature. Many graduate students in mathematics have no sufficient background in physics apart of elementary undergraduate courses and the relation to physics of the mathematical objects and notions studied in their graduate classes is often not emphasized enough. The goal of this course is to fill this gap.

The main emphasis will be given not to details of physics but to mathematical ideas and tools needed in developing these physical theories/equations so that a student will understand the motivation and the origin for introducing and studying this mathematical tools. The problems



on rigorous mathematical formulations of some physical theories (such as quantum field theory and functional integrals) will be discussed as well.

We will start with Classical Mechanics by relating it to Calculus of Variation (the least action principle), Symplectic Geometry (via developing the Hamiltonian formalism), Lie group/ Lie algebra theory (via the theory of Rigid body motion), First integrals/ Conservation laws, Noether theorems, Liuoville integrability, and action-angle theorem will be discussed here as well

Then we will discuss elements of special relativity, classical electrodynamics (including Maxwell equation via differential forms and first discussions of gauge transformation), general relativity (very brief, a glimpse on derivation of Einstein field equation).

Finally, we will discuss basic of quantum mechanics, elements of relativistic quantum mechanics (Klein-Gordon and Dirac equations) and very basic quantum field theory (free quantum fields, Wightman axioms, some elements of perturbative field theory and Feynman diagrams).

At the end of the class every student will choose a topic for one-hour presentation from the broad list of topic I will provide or a topic of their own interest from modern physics which strongly uses mathematics.

Course Prerequisites

Multivariable Calculus (MATH 221, MATH 251 or equivalent), Linear Algebra (MATH 304, MATH 309, MATH 311, MATH 323, Math 640, or equivalent).

Course Learning Outcomes

Upon successful conclusion of this course students should be able to:

- derive all major equations in classical and modern physics;
- give physical motivation for main mathematical objects in several mathematical disciplines such
 as symplectic geometry, Hamiltonian dynamics, calculus of variation, differential geometry, and
 mathematical gauge theory, from theory of Lie groups, Lie algebras, Clifford algebras, Hopf
 algebras/ quantum groups, and their representations, spectral theory of differential operators,
 theory of operator algebras, theory of characteristic classes of vector bundles, and knot theory;
- apply the tools and techniques from the mathematical theories mentioned above for solving concrete problems in classical and modern physics;
- conduct interdisciplinary research in mathematical physics;
- identify problems of rigorous mathematical formulation of some physical theories (such as quantum field theory, functional integrals, renormalization).

Textbook and/or Resource Materials:



- The main textbook for the part of the course related to Classical Mechanics is
 - Vladimir Arnol'd, Mathematical methods of classical mechanics. Translated from the 1974 Russian original by K. Vogtmann and A. Weinstein. Corrected reprint of the second (1989) edition. Graduate Texts in Mathematics, 60. Springer-Verlag, New York, [1989]. xvi+516 pp
- The two main resources for the rest of the course are
 - Gerald B. Folland, Quantum field theory. A tourist guide for mathematicians.
 Mathematical Surveys and Monographs, 149. American Mathematical Society, Providence, RI, 2008. xii+325 pp
 - o **I. D. Lawrie**, A Unified Grand Tour of Theoretical Physics, Institute of Physics Publishing, 2nd edition, 2002.

Grading Policy

Your final grade will be determined by your performance on the homework and the final presentation. The grade ingredients are:

Activity	• %
Bi-weekly Homework Assignments	70%: It will be usually posted on Tuesdays with due date in 2 weeks. Homework submission will be via Canvas
• Final presentation	30%: In the beginning of November it will be assigned to you in advanced on a topic which continues some topic discussed in a class or on a new topic which was not covered in class. The presentation will be in the final exam period, i.e. Dec 6-Dec 13. The schedule will be determined later.
Total:	• 100%

Grading Scale

• Range	Grade
• 90 -100 %	• A
• 80- 89 %	• B
• 70-79 %	• C
• 60-69 %	• D
• 0-59 %	• F

Grade complaints:



If you think a homework was graded incorrectly you have one week from the time the graded assignment was returned to you to bring the issue to the instructor's attention. No complaints after that time will be considered.

Exams: There will not be exams in this class.

Late Work Policy

Late work (e.g., submitting a homework assignment after the established deadline). will NOT be accepted unless a student has a University approved reason and contact me within two working days of the missed assignment. If a student is unable to make the final presentation in the scheduled day, he must notify me two working days before to reschedule the date of this presentation.

Appeal Policy

Students have one week upon the return of individual grades to notify the instructor of any inaccuracies in their graded work. Students should bring all grade disputes to their instructor in an individual Zoom meeting. Due to FERPA privacy issues, grade disputes will not be discussed over email or in the classroom.

Working with Friends

Working together on homework is fine and encouraged, but each of you must write up your own solutions in your own words, notation and/or symbols and write the names of your collaborators at the top left corner of your homework. Copying a solution from a source and referencing the source is still considered a violation of academic integrity because you are submitting work for a grade that is not your own work. It is NOT permissible to discuss any aspect of any quiz, test or examination until ALL students have completed it. The penalties for violating this policy will range from an F on an assignment or test, to failing in the course.

Attendance

Attendance is essential to complete this course successfully.

Excused Absences: University student rules concerning excused and unexcused absences, as well as makeups, can be found at http://student-rules.tamu.edu/rule07. Make-up exams and quizzes or late homework, writing assignments will NOT be allowed unless a University approved reason is given to me in writing. Notification before the absence is required when possible. Otherwise (e.g., accident, or emergency), you must notify me within two business days of the missed exam, quiz, or assignment to arrange a makeup.

Other Important Dates: Aug 25 (last day to add or drop a course), Sep 4 (Labor Day), Octr. 9 – Oct. 10 (Fall break), Nov 15 (Q-drop deadline), Nov 22 (Reading Day), Nov 23&24 (no class-- Thanksgiving),



Tentatve Course Schedule

WEEK	Торіс
WEEKS 1-2	Least action principle, Euler-Lagrange Equation, Legendre Transform, Hamiltonian Equation (Arnol'd Chapter 3), First integrals/ Conservation laws, Noether theorems, D'Alambert theorem (Arnol'd chapter 4)
WEEKS 2-3	Motion of Rigid bodies (Arnol'd chapter 6)
WEEKS 3-4	Hamiltonian Formalism {Review of differential forms and tensors, cotangent bundle symplectic manifold, Poison brackets, integral invariant of Poincare -Cartan, Hamilton-Jacobi equations, Arnold, chapters 8, 9)
WEEKS 4-5	Liouville integrable systems, action-angle variables, averaging (Arnol'd, Chaoter 10)
WEEK 6	Introduction to special relativity (Lawrie, sections 2.0.1, 3.5, Folland, section 2.3)
WEEK 7	Classical electromagnetism (Lawry . section 3.6, Folland, section 2.4)
WEEKS 8-9	Elements of general relativity (Lawry, Chapter 4)
WEEK 10-11	Basic Quantum Mechanics (Lawry Chapter 5, Folland Chapter 4)
WEEKS 12-13	Relativistic Quantum Mechanics (Folland, Chapter 4, Lawry, sections 7.3-7.6)
WEEKS 14-15	Basic Quantum Field theory (free quantum fields, Wightman axioms, some elements of perturbative field theory and Feynman diagrams, Folland, Chapter 5-6, Lawry, chapter 6, sections 7.1&7.2)

University Policies

This section outlines the university level policies that must be included in each course syllabus. The TAMU Faculty Senate established the wording of these policies.

NOTE: Faculty members should not change the written statements. A faculty member may add separate paragraphs if additional information is needed.

Attendance Policy

The university views class attendance and participation as an individual student responsibility. Students are expected to attend class and to complete all assignments.

Please refer to <u>Student Rule 7</u> in its entirety for information about excused absences, including definitions, and related documentation and timelines.

Makeup Work Policy

Students will be excused from attending class on the day of a graded activity or when attendance contributes to a student's grade, for the reasons stated in Student Rule 7, or other reason deemed appropriate by the instructor.



Please refer to <u>Student Rule 7</u> in its entirety for information about makeup work, including definitions, and related documentation and timelines.

Absences related to Title IX of the Education Amendments of 1972 may necessitate a period of more than 30 days for make-up work, and the timeframe for make-up work should be agreed upon by the student and instructor" (Student Rule 7, Section 7.4.1).

"The instructor is under no obligation to provide an opportunity for the student to make up work missed because of an unexcused absence" (<u>Student Rule 7, Section 7.4.2</u>).

Students who request an excused absence are expected to uphold the Aggie Honor Code and Student Conduct Code. (See <u>Student Rule 24</u>.)

Academic Integrity Statement and Policy

"An Aggie does not lie, cheat or steal, or tolerate those who do."

"Texas A&M University students are responsible for authenticating all work submitted to an instructor. If asked, students must be able to produce proof that the item submitted is indeed the work of that student. Students must keep appropriate records at all times. The inability to authenticate one's work, should the instructor request it, may be sufficient grounds to initiate an academic misconduct case" (Section 20.1.2.3, Student Rule 20).

Texas A&M at College Station

You can learn more about the Aggie Honor System Office Rules and Procedures, academic integrity, and your rights and responsibilities at <u>aggiehonor.tamu.edu</u>.

Texas A&M at Galveston

You can learn more about the Honor Council Rules and Procedures as well as your rights and responsibilities at tamuq.edu/HonorSystem.

Texas A&M at Qatar

You can learn more about academic integrity and your rights and responsibilities at Texas A&M University at Qatar by visiting the <u>Aggie Honor System</u> website.

Americans with Disabilities Act (ADA) Policy

Texas A&M University is committed to providing equitable access to learning opportunities for all students. If you experience barriers to your education due to a disability or think you may have a disability, please contact the Disability Resources office on your campus (resources listed below) Disabilities may include, but are not limited to attentional, learning, mental health, sensory, physical, or chronic health conditions. All students are encouraged to discuss their disability related needs with Disability Resources and their instructors as soon as possible.



Texas A&M at College Station

Disability Resources is located in the Student Services Building or at (979) 845-1637 or visit disability.tamu.edu.

Texas A&M at Galveston

Disability Resources is located in the Student Services Building or at (409) 740-4587 or visit tamug.edu/counsel/Disabilities.

Texas A&M at Qatar

Disability Services is located in the Engineering Building, room 318C or at +974.4423.0316 or visit https://www.gatar.tamu.edu/students/student-affairs/disability-services.

Title IX and Statement on Limits to Confidentiality

Texas A&M University is committed to fostering a learning environment that is safe and productive for all. University policies and federal and state laws prohibit gender-based discrimination and sexual harassment, including sexual assault, sexual exploitation, domestic violence, dating violence, and stalking.

With the exception of some medical and mental health providers, all university employees (including full and part-time faculty, staff, paid graduate assistants, student workers, etc.) are Mandatory Reporters and must report to the Title IX Office if the employee experiences, observes, or becomes aware of an incident that meets the following conditions (see <u>University Rule 08.01.01.M1</u>):

- The incident is reasonably believed to be discrimination or harassment.
- The incident is alleged to have been committed by or against a person who, at the time of the incident, was (1) a student enrolled at the University or (2) an employee of the University.

Mandatory Reporters must file a report regardless of how the information comes to their attention — including but not limited to face-to-face conversations, a written class assignment or paper, class discussion, email, text, or social media post. Although Mandatory Reporters must file a report, in most instances, a person who is subjected to the alleged conduct will be able to control how the report is handled, including whether or not to pursue a formal investigation. The University's goal is to make sure you are aware of the range of options available to you and to ensure access to the resources you need.

Texas A&M at College Station

Students wishing to discuss concerns in a confidential setting are encouraged to make an appointment with Counseling and Psychological Services (CAPS).

Students can learn more about filing a report, accessing supportive resources, and navigating the Title IX investigation and resolution process on the University's <u>Title IX webpage</u>.

Texas A&M at Galveston



Students wishing to discuss concerns in a confidential setting are encouraged to make an appointment with the Counseling Office in the Seibel Student Center, or call (409)740-4587. For additional information, visit tamuq.edu/counsel.

Students can learn more about filing a report, accessing supportive resources, and navigating the Title IX investigation and resolution process on the Galveston Campus' Title IX webpage.

Texas A&M at Qatar

Texas A&M University at Qatar students wishing to discuss concerns in a confidential setting are encouraged to visit the <u>Health and Wellness</u> website for more information.

Students can learn more about filing a report, accessing supportive resources, and navigating the Title IX investigation and resolution process on the University's Title IX webpage.

Statement on Mental Health and Wellness

Texas A&M University recognizes that mental health and wellness are critical factors that influence a student's academic success and overall wellbeing. Students are encouraged to engage in healthy self-care by utilizing available resources and services on your campus

Texas A&M College Station

Students who need someone to talk to can contact Counseling & Psychological Services (CAPS) or call the TAMU Helpline (979-845-2700) from 4:00 p.m. to 8:00 a.m. weekdays and 24 hours on weekends. 24-hour emergency help is also available through the 988 Suicide & Crisis Lifeline (988) or at 988lifeline.org Links to an external site..

Texas A&M at Galveston

Students who need someone to talk to can call (409) 740-4736 from 8:00 a.m. to 5:00 p.m. weekdays or visit tamug.edu/counsel for more information. For 24-hour emergency assistance during nights and weekends, contact the TAMUG Police Dept at (409) 740-4545. 24-hour emergency help is also available through the 988 Suicide & Crisis Lifeline (988) or at 988lifeline.org Links to an external site..

Texas A&M at Qatar

Texas A&M University at Qatar students wishing to discuss concerns in a confidential setting are encouraged to visit the <u>Health and Wellness</u> website for more information.

Campus-Specific Policies

Texas A&M at Galveston

Classroom Access and Inclusion Statement

Texas A&M University is committed to engaged student participation in all of its programs and courses and provides an accessible academic environment for all students. This means that our classrooms, our virtual spaces, our practices and our interactions are as inclusive as possible and we work to provide a



welcoming instructional climate and equal learning opportunities for everyone. If you have an instructional need, please notify me as soon as possible.

The Aggie Core values of respect, excellence, leadership, loyalty, integrity and selfless service in addition to civility, and the ability to listen and to observe others are the foundation of a welcoming instructional climate. Active, thoughtful and respectful participation in all aspects of the course supports a more inclusive classroom environment as well as our mutual responsibilities to the campus community.

The following statements below are optional. Leave as is to include, or delete if preferred. Either way, delete this note.

Statement on the Family Educational Rights and Privacy Act (FERPA)

FERPA is a federal law designed to protect the privacy of educational records by limiting access to these records, to establish the right of students to inspect and review their educational records and to provide guidelines for the correction of inaccurate and misleading data through informal and formal hearings. Currently enrolled students wishing to withhold any or all directory information items may do so by going to howdy.tamu.edu and clicking on the "Directory Hold Information" link in the Student Records channel on the MyRecord tab. The complete FERPA Notice to Students and the student records policy is available on the Office of the Registrar webpage.

Items that can never be identified as public information are a student's social security number, citizenship, gender, grades, GPR or class schedule. All efforts will be made in this class to protect your privacy and to ensure confidential treatment of information associated with or generated by your participation in the class.

Directory items include name, UIN, local address, permanent address, email address, local telephone number, permanent telephone number, dates of attendance, program of study (college, major, campus), classification, previous institutions attended, degrees honors and awards received, participation in officially recognized activities and sports, medical residence location and medical residence specialization.

College and Department Policies

College and departmental units may establish their own policies and minimum syllabus requirements. As long as these policies and requirements do not contradict the university level requirements, colleges and departments can add them in this section. Please remove this section if not needed.